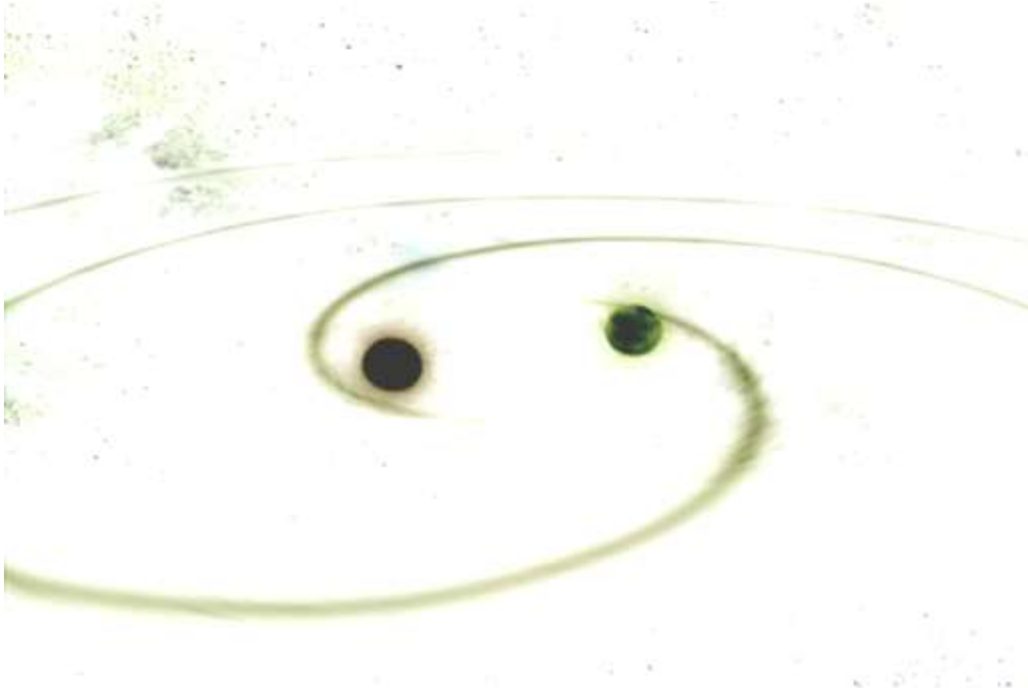


TianQin – Space-borne gravitational wave detector

SPACECRAFT NAVIGATION AND MISSION SIMULATION

December 9, 2015 - Prepared by Viktor T. Toth

A PERSPECTIVE

- Precision navigation
- End-to-end mission simulation

A NEW TYPE OF MISSION

- Three (or more) spacecraft form a single science instrument
- Test mass orbits are, in effect, the science observable
- GW signal is extremely faint: $h \sim \Delta L/L \sim 2(GM/c^2r)(GM/c^2R) \lesssim 10^{-20}$

THE CHALLENGE

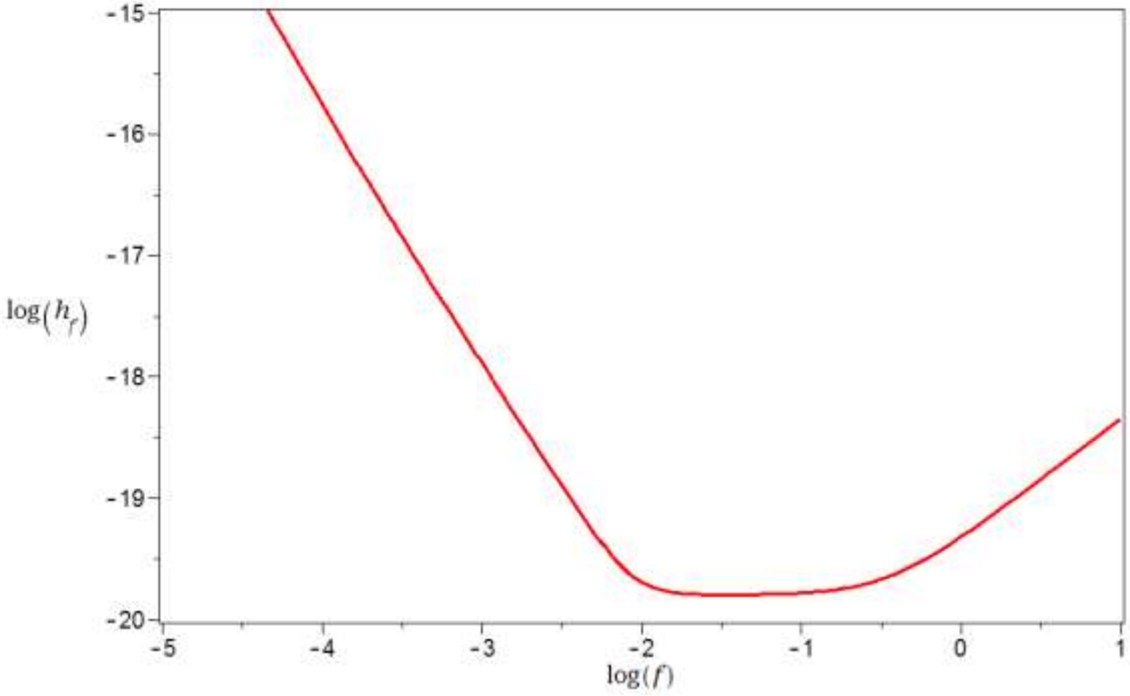
- Precision-navigate a constellation
- Achieve sufficient navigational accuracy
- Disentangle gravitational wave contributions from other (gravitational and nongravitational) effects
- Difficult to test in a terrestrial (1g) laboratory environment
- LISA Pathfinder tests key technologies but not TDI
- Modeling and simulation are essential

REQUIREMENTS

- Acceleration accuracy
- Positional accuracy
- Unequal arms vs. laser noise

SENSITIVITY GOAL

- Sensitivity goal at SNR = 1 dB



$$h_f = \frac{2}{\sqrt{R(2\pi f)}}\sqrt{\frac{S_x}{L_0^2} + \frac{S_a}{(2\pi f)^4 L_0^2}\left(1 + \frac{10^{-4}\mathrm{Hz}}{f}\right)}$$

$$R(w) \approx \frac{8}{15}\left[1 + \left(\frac{wL_0}{0.41\pi c}\right)^2\right]^{-1}$$

$$\sqrt{S_x} = 10^{-12}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$$

$$\sqrt{S_a} = 3.1 \times 10^{-15}\ \mathrm{m/s^2}/\sqrt{\mathrm{Hz}}$$

$$L_0 = \sqrt{3} \times 10^5\ \mathrm{km}$$

Near $10^{-2}\mathrm{Hz}$, the contributions of S_a and S_x are similar in magnitude.

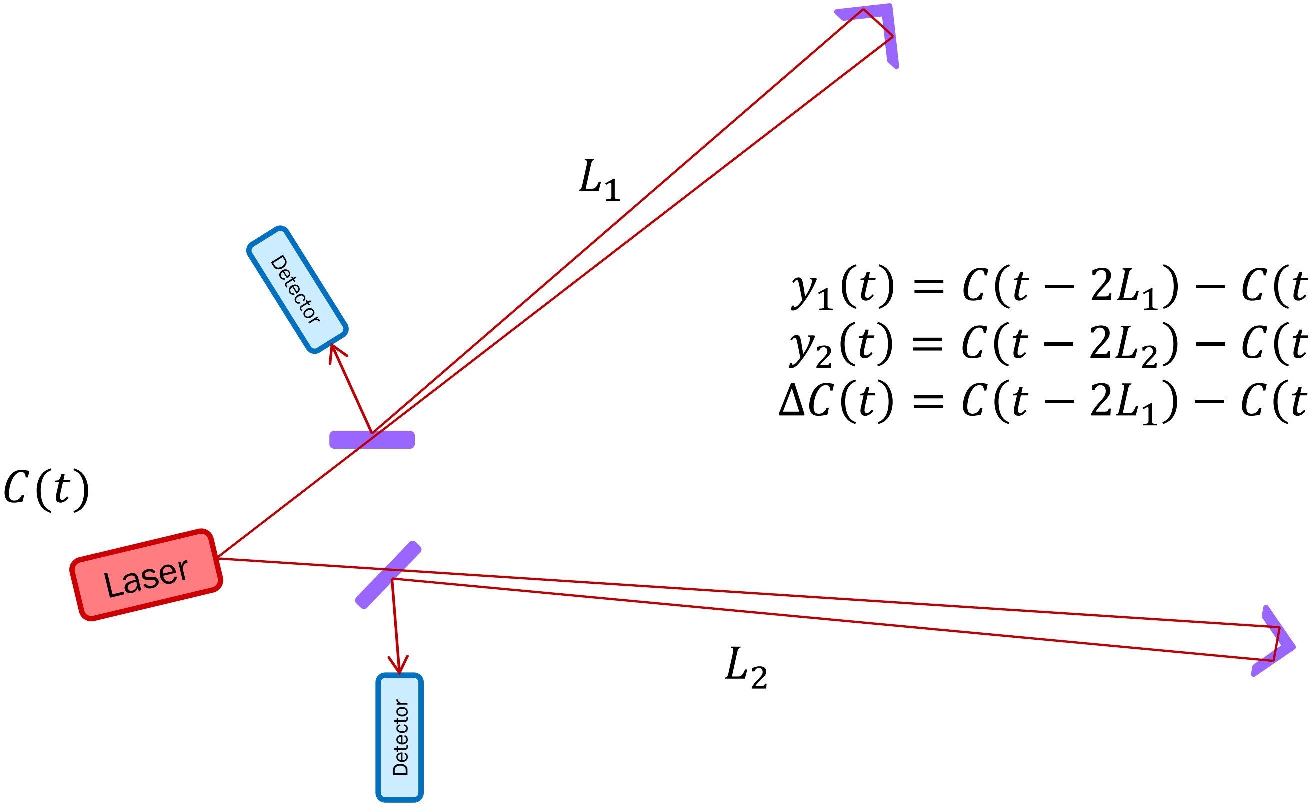
ACCELERATION ACCURACY

- Non-gravitational noise compensation at the level of $10^{-12}\ \mathrm{m/s^2/\sqrt{Hz}}$
- Test mass residual acceleration of $10^{-15}\ \mathrm{m/s^2/\sqrt{Hz}}$
- For comparison, the unmodeled (anomalous) acceleration of the Pioneer 10 and 11 spacecraft was $\sim 10^{-9}\ \mathrm{m/s^2}$

POSITIONAL ACCURACY

- Test mass positional accuracy of $1\ \mathrm{pm}/\sqrt{\mathrm{Hz}}$
- For comparison, GRACE Follow-On has an accuracy requirement of a few nm

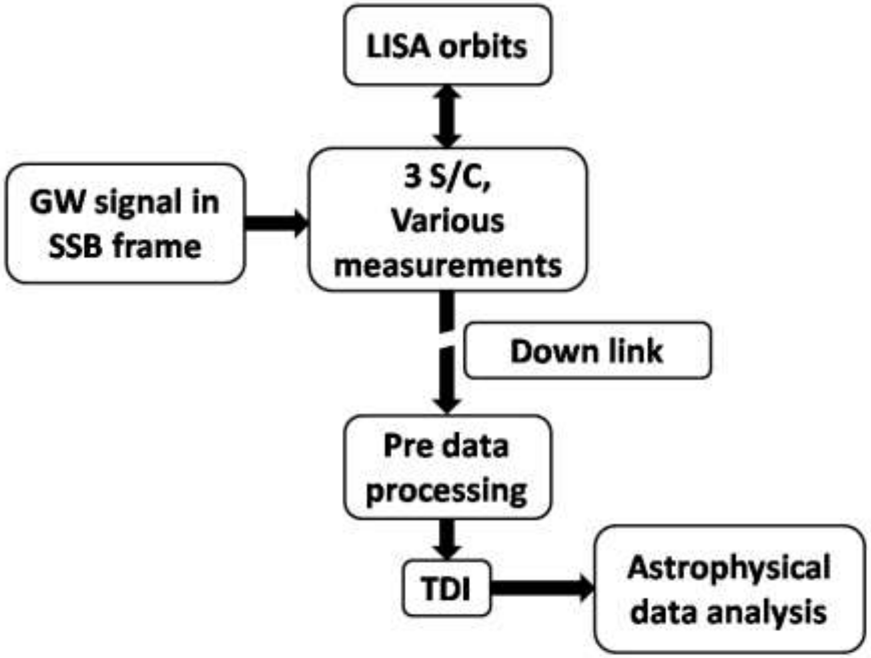
UNEQUAL ARMS AND TDI



$$y_1(t) = C(t - 2L_1) - C(t) + h_1(t) + n_1(t)$$
$$y_2(t) = C(t - 2L_2) - C(t) + h_2(t) + n_2(t)$$
$$\Delta C(t) = C(t - 2L_1) - C(t - 2L_2)$$

- Raw data referred to unsynchronized clocks with individual drift and jitter
- Requires precise knowledge of arm lengths and longitudinal velocities

LISA(-LIKE) DATA PROCESSING



LISA orbits

GW signal in SSB frame

3 S/C, Various measurements

Down link

Pre data processing

TDI

Astrophysical data analysis

Yan Wang, *On inter-satellite laser ranging, clock synchronization and gravitational wave data analysis*, PhD thesis (2014)

- But where does the orbital data come from?

NAVIGATION – ORBITAL RECONSTRUCTION

- “Live” navigation vs. reconstruction of orbits
- Kalman-filtering is used to refine orbital estimates of flying spacecraft
- Nonlinear least squares estimator can be used for orbital reconstruction

On the following slides, I shall use “navigation” to describe both live navigation and orbital reconstruction.

TWO NAVIGATIONAL PROBLEMS

- Navigating the spacecraft with the requisite positional accuracy requires precise knowledge of small nongravitational forces
- Navigating the test masses requires very accurate knowledge of the gravitational field
- If test masses are not 3D drag-free, they are not following geodesics
- Absolute positions may not be important; relative distances and velocities essential for TDI

ZONAL HARMONICS OF THE EARTH

- Zonal harmonics represent a significant potential noise source in the critical frequency range

$$U_E = \frac{GM_E}{r}\left\{1 - \sum_{l=2}^{\infty}\left[\left(\frac{R_E}{r}\right)^l \sum_{k=0}^{l} P_{lk}(\cos\theta)(C_{lk}\cos k\phi + S_{lk}\sin k\phi)\right]\right\}$$

RELATIVISTIC CONTRIBUTIONS

- $\frac{d^2\mathbf{r}}{dt^2} = \frac{\nabla U_i}{|\mathbf{r}_i - \mathbf{r}|}[A_i(\mathbf{r}_i - \mathbf{r}) + \mathbf{B}_i]$
- $A_i = 1 - \frac{1}{c^2}\left\{2(\beta+\gamma)\sum_j \frac{\mu_j}{|\mathbf{r}_j - \mathbf{r}|} + \gamma v^2 + (1+\gamma)\mathbf{v}\cdot\mathbf{v}_i - \frac{3}{2}\left[\frac{(\mathbf{r}-\mathbf{r}_i)\cdot\mathbf{v}_i}{|\mathbf{r}_i - \mathbf{r}|}\right]^2\right\}$
- $\mathbf{B}_i = \frac{1}{c^2}\{(\mathbf{r}-\mathbf{r}_i)\cdot[(2+2\gamma)\mathbf{v} - (1+2\gamma)\mathbf{v}_i]\}(\mathbf{v}-\mathbf{v}_i)$

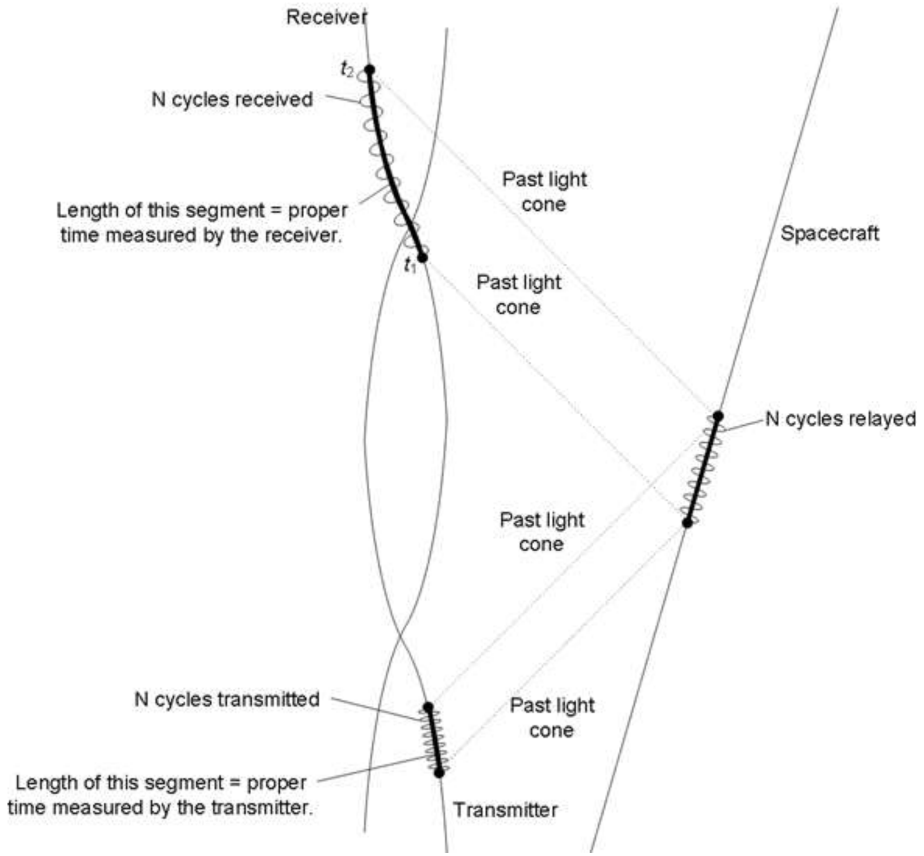
T. D. Moyer, *Formulation for Observed and Computed Values of Deep Space Network Data Types for Navigation*, John Wiley & Sons (2005)

SMALL FORCES

- Drag (collisions with dust, upper atmosphere, etc.)
- Solar radiation pressure ($4.6\ \mu\mathrm{N/m^2}$)
- Maneuvers
- Outgassing from thrusters
- Outgassing from other equipment
- Outgassing from surface coating materials ($\sim \mathrm{nN/m^2}$, Schläppi et al., 2012)
- Radio and laser beam recoil force ($\sim 3.3\ \mathrm{nN/W}$)
- Thermal radiation recoil force

NAVIGATION

- Radio-metric or optical
- Doppler or range



SIGNAL PROPAGATION

- Gravitational (Shapiro) delay
- Charged particles (solar wind)
- Ionosphere
- Wet troposphere

SIGNAL RECEPTION

- Earth precession and nutation
- Earth tides
- Continental drift

EXISTING KNOWLEDGE

- The navigational problem is well-understood
  - VLBI
  - GPS
  - Precision deep space navigation (Pioneer)

- Emphasis must be on contributions in the sensitive frequency range (mHz)

THE SIMULATION CHALLENGE

- Orbital simulation is (relatively) easy
  - We treat the S/C as a point test particle subject to a range of forces, and we simulate signal propagation

- What else do we wish to simulate for a successful mission?

FAILURE ANALYSIS

- Imagine the mission fails. What are some possible causes?
  - Test masses are not traveling along predictable orbits. Specifically, they are perturbed by unmodeled forces in the critical frequency range
  - Unmodeled temperature fluctuations affect measurements either by changing the optical path length or by impacting optical equipment (filters)

WHAT MUST BE MODELED?

- Orbits (obviously)
- Mechanical and thermal behavior
  - Especially the mechanical and thermal behavior of spacecraft components along the light path
- Optical and electrical behavior of the laser interferometer system

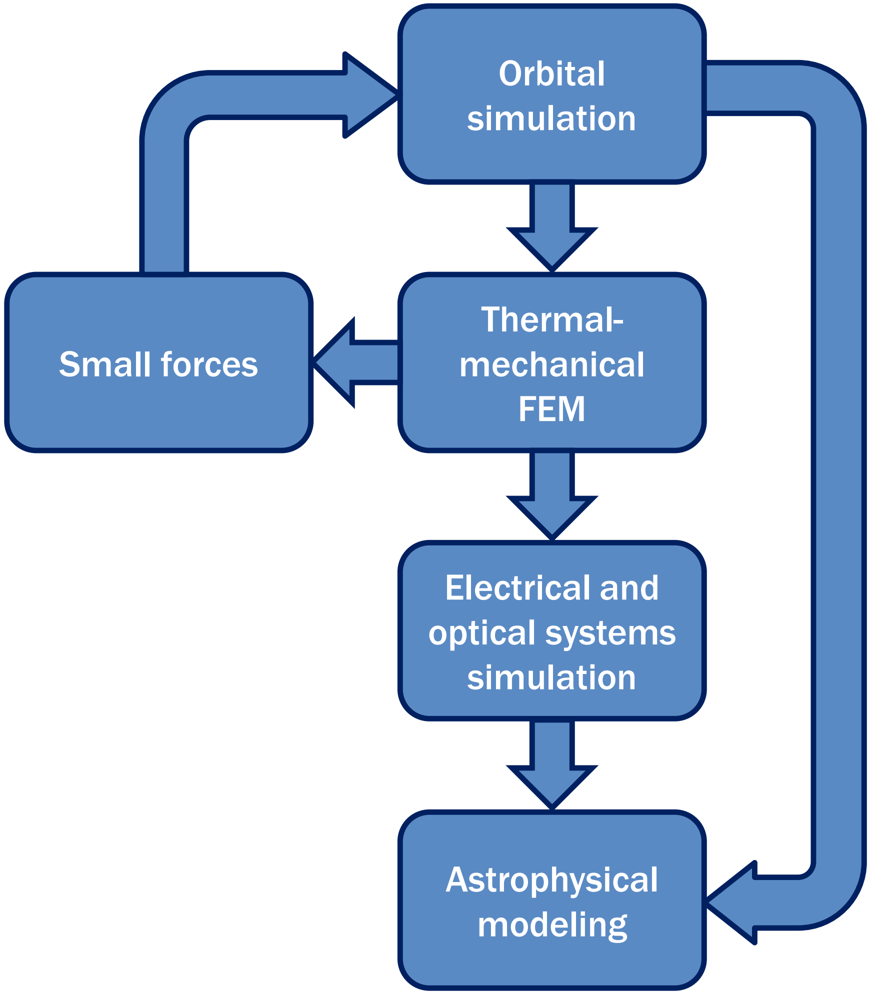
THE COMPUTATIONAL CHALLENGE

- Simultaneous simulation of interdependent systems can increase computational requirements by orders of magnitude
- Supercomputers are great to have but they are no excuse for bad algorithms that can overwhelm even the best hardware

A SIMULATION STRATEGY

- Simulation tasks can be performed independently and iteratively:
  1. orbital simulation of the S/C as a whole can provide input data to compute thermal exposure to sunlight, thermal radiation from the Earth, and other sources of heat;
  2. Finite element model of the S/C can provide detailed estimates with high temperature resolution of the spacecraft's thermal behavior;
  3. Thermal estimate can be used to refine the orbital estimate by incorporating very small thermal recoil forces;
  4. The thermal estimate can be used as input data for the optical and electrical simulation.

ITERATIVE SIMULATION



SOFTWARE DEVELOPMENT AND TESTING

- Project is both straightforward and challenging
- Uncommon accuracy (beyond IEEE 64-bit)
- No test cases
- Critical to successful mission design
- Strict formal methodology essential; should include validation (perhaps borrowing formal validation processes from avionics or medical device software)

THANK YOU

- Questions?